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DISCUSSION OF PROCEEDINGS - SEPARATES

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HYDRAULICS DIVISION

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DISCUSSION OF NONLINEAR ELECTRICAL ANALOGY FOR PIPE NETWORKS PROCEEDINGS-SEPARATE NO. 139

CHARLES L. BARKER,[13] A. M. ASCE.—The theory and principles of operation for a new engineering tool are presented in this paper—a tool that will become of increasing value as engineers become aware of its usefulness and capabilities. The author has not mentioned the many applications of the network analyzer. Perhaps he left this phase to the imagination of the reader.

At the State College of Washington, at Pullman, problems of air, gas, oil, and water have been worked on it, and, as is the case in many new tools, the analyzer has been able to do many things that were not even considered in the original planning.

On a problem of a distribution system for a city of about 50,000, extensive changes were contemplated. These involved increasing pipe sizes in the present system to meet heavier loads, extension of present mains to take care of population growth, and, finally, studying the effect of additional sources. Added to these was the problem of adequate fire protection. In 21 hr of analyzer operation, twenty-four pressure contour maps were prepared presenting the various combinations of the foregoing problems. The expected pressure for the given flow conditions was given on each map. This pressure was equivalent to a solution of a thirty-five-loop system by the Hardy Cross method twenty-four times. In fact, the work was more than that because when each study was started, preliminary tests of pressure were made for the flow conditions assumed. If the pressures were unsatisfactory, pipe changes were made by changing the nonlinear resistors, and then the test began again.

In another study it was planned to remove an undesirable source from the system and to supply the system from a reservoir to be filled in off-peaks operation. The system was placed on the analyzer, pipes were changed to give adequate pressure and fire protection, and, finally, the height of the reservoir was determined. The analyzer time was 8 hr.

One problem of maximum fire flow on a system was studied. A load representing a fire load was placed at different points in the system. The size of this load was increased until the pressure at the outlet was as low as was felt to be satisfactory, thus giving the maximum fire load possible.

Studies of the effect of pipe roughness on system-pumping costs can be made easily, and the effect of changing the roughness value is thus made immediately apparent. All the foregoing comments refer to the flow of water. Similar studies can be made on other fluids.

Mr. McIlroy has given the engineer a new tool—one that he must learn to use before its value can be fully appreciated—and engineers owe him a vote of thanks.

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Malcolm S. McIlroy[14].—The description, by Mr. Barker, of the solution of four problems by use of the nonlinear analyzer at the State College of Washington, constitutes a valuable addition to the paper. Similar studies of distribution systems for water, steam, and gas have been conducted on the analyzers at the Midwest Research Institute, in Kansas City, Mo., and at Cornell University, at Ithaca, N. Y. Studies have also been made of the performance of a storm sewer system and of the circulation of ventilating air in a coal mine. Investigations in the field of gas distribution, by use of large nonlinear analyzers, will be undertaken at Columbus, Ohio, St. Louis, Mo., and Newark, N. J.

As Mr. Barker indicates, the use of the analyzer as an aid to design can best be appreciated after actual experience. If various conditions are imposed on a network in a logical sequence, the results of each test suggest alternatives that may be investigated at once, leading to a judicious solution to the problem. The elimination of extensive computations greatly extends the possibility of undertaking investigations that might otherwise be prohibited by cost and time requirements.

Since the original paper was written, extensions of the method to satisfy square-law fluid-flow equations have been published.[15] This method has also been used to investigate the flow of compressible fluids.[16] Convenient tables of constants applying to water-distribution have also been made available.[17]

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[14] Prof. of Electrical Eng., Cornell Univ., Ithaca, N. Y.; and Consultant to the Standard Electric Time Co., Springfield, Mass.

[15] "Gas Pipe Networks Analyzed by Direct-Reading Electric Analogue Computer," by Malcolm S. McIlroy, *Publication DMC-52-10*, Am. Gas Assn., April, 1952.

[16] "Steam-Distribution Systems Analyzed by Nonlinear Electrical-Analogy Method," by Malcolm S. McIlroy and Chao K. Chow, *Proceedings*, National District Heating Assn., Vol. XLIII, 1952.

[17] "Water-Distribution Systems Studied by a Complete Electrical Analogy," by Malcolm S. McIlroy, *Journal*, New England Water Works Assn., Vol. LXV, No. 4, 1951, p. 299.

DISCUSSION OF ELECTRICAL ANALOGIES AND ELECTRONIC COMPUTERS: SURGE AND WATER HAMMER PROBLEMS PROCEEDINGS-SEPARATE NO. 146

E. B. STROWGER[49].—The basic dynamics of the performance of hydroelectric generating units during transient effects have been well established for several years. Computations of water hammer pressures can be made quickly and accurately, as can computations of speed rise for cases in which the unit drops its load and is freed from the system. Surge problems involving surge tank operation may also be readily solved.[50,51] Where the problem includes the electrical system, or part of it, as well as the equipment between the water intake and the station bus, some research may be necessary. Apparently, this is the object of the author's investigation—the solving of transients involving electrical systems by means of the electronic computer.

In almost any problem involving a transient, if the basic relations among the various physical components of the system are known, the solving of a set of simultaneous equations by arithmetic integration is all that is necessary for an accurate solution. The presence of nonlinearities which hamper a general solution does not hamper the application of arithmetic integration, and an ordinary slide rule—preferably a 20-in. one—is all that is required. The use of as many as eight or nine simultaneous equations for problems involving a double-ported differential tank or a spilling tank is not uncommon, although the usual problems involve only six or seven equations. Results of field tests of tank performance, even under conditions that are complicated, have shown good agreement with computed performance curves. The design of the surge tank installation at Fort Peck Dam in Montana included three interconnected orifice-type tanks, one on each of the three 14-ft penstocks that in turn were supplied by a tunnel 24.67 ft in diameter. In addition, there was a control shaft (or simple tank) of 50-ft diameter on the supply tunnel, half way between the powerhouse and the intake. Computations by arithmetic integration of the effect of the control shaft on the functioning of the surge tanks were checked by model tests that showed very satisfactory agreement.[52]

To the writer's knowledge, the simple surge tank is no longer used in new installations and he assumes that the author has used it to insure simplicity in his example. For this purpose, the author has selected the Tallulah Falls tank described by E. Lauchli.[48] Fig. 17 reproduces the experimental surge curve given by Mr. Lauchli as the test record for a change in velocity of from 6.13 ft per sec to zero ft per sec caused as follows:

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[49] Chf. Hydr. Engr., Niagara Mohawk Power Corp., Buffalo, N. Y.

[50] "Speed Changes of Hydraulic Turbines for Sudden Changes of Load," by E. B. Strowger and S. Logan Kerr, *Transactions*, ASME, Vol. 48, 1926, p. 209.

[51] "Arithmetic Integration Applied to Surge Tank Problems," by E. B. Strowger, Chapter 35 B in "Hydroelectric Handbook," by W. P. Creager and J. D. Justin, John Wiley & Sons, Inc., New York, N. Y., 1950.

[52] "Model Study of Hydraulic Characteristics of Power Tunnel, Fort Peck Dam," *Technical Memorandum No. 185-1*, U. S. Waterways Experiment Station, Vicksburg, Miss.

"The relief valves of the turbine were held open with chain blocks and at a given time they were left to close rapidly. The average total closure of the valves took place in less than 40 seconds, the relief valves closing rapidly first during the first 25 to 30 seconds and then more slowly ***."[45]

Point A1 in Fig. 17 represents the surge height at the end of a quarter period as determined by the electronic computer. The lack of agreement of results is explained by the author as "*** caused primarily by the slow rate at which the demand was reduced."

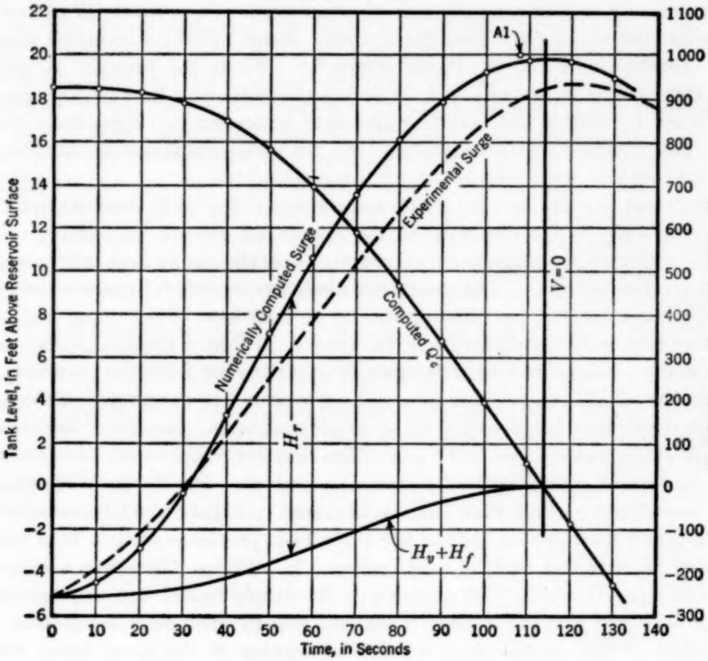

FIG. 17

The writer has computed the surge curve for this particular problem by arithmetic integration as shown by Fig. 17, assuming a full closure from a discharge of 925 cu ft per sec to a discharge of 0 cu ft per sec. (6.13 ft per sec to 0 ft per sec) in 40 sec. The close agreement between point A1 and the maximum surge of the numerically computed curve is apparent. The small differences between the computed results and the test results suggest a slight change in the initial discharge, final discharge, or load-rejection time. In Fig. 17, H_v represents velocity head, H_f is friction loss, and H_r designates the retarding head.

One advantage of arithmetic integration as a method of computation is that it produces the complete surge curve which may be used as a check on the

over-all solution. This check is made by plotting the relationship between retarding head and time, and measuring the area under the curve from zero time to the point at which the velocity of the conduit becomes zero. This area, in units of foot-seconds, should equal the initial momentum of the water in the conduit, $\frac{L_c V}{g}$ in foot-seconds. The area of the retarding head-time curve is

$$\int_{t=0}^{t=T_m} H_r \, dt \qquad (49)$$

in which T_m is the time at which the velocity becomes zero. This area, shown in Fig. 17, is bounded by the surge curve, the curve of head recovery, and the vertical line representing the time of zero velocity. This impulse area, in foot-seconds, is 1,280 compared with an initial momentum in the conduit of

$$L_c \frac{V}{g} = \frac{6{,}666 \times 6.13}{32.2} = 1{,}270 \text{ ft-sec} \qquad (50)$$

Another advantage of the arithmetic integration method becomes apparent if the tank is not cylindrical throughout its useful range. In this case it is comparatively easy to change the constants of the installation as the integration process advances.

The writer concludes that the solution of transients in water hammer problems by means of the electronic computer lacks some of the advantages of the arithmetic integration method of solution.

George R. Rich,[53] M. ASCE.—The penetrating analysis of the science of hydraulic transients found in this paper is admirable. Naturally, the first impression created by Fig. 6 is one of alarm for the safety of surge tank and pressure conduit systems constructed long before the pioneering of Daniel Gaden in the theory of governing stability; yet the operating records of many successful commercial installations appear to belie the existence of any catastrophic super-pressures or sub-pressures caused by unanticipated resonant conditions. For a possible explanation of this seeming disparity between theory and practice, the case of Fig. 6(a) will be considered. This is the simple pressure conduit without surge tank under the influence of a permanently sustained small oscillation of the turbine gates.

The discusser's independent analytical work indicates that, under the assumed "hunting" of the turbine gates, nodes will occur at values of x (measured from the reservoir as origin) equal to 0, $\frac{a\,T_0}{2}$, $a\,T_0$, $\frac{3\,a\,T_0}{2}$, $2\,a\,T_0$, $\frac{5\,a\,T_0}{2}$, . . . $\frac{n\,a\,T_0}{2}$; and maxima (and minima) may be expected at x equal to $\frac{a\,T_0}{4}$, $\frac{3\,a\,T_0}{4}$, $\frac{5\,a\,T_0}{4}$, $\frac{7\,a\,T_0}{4}$, . . . $\frac{(2\,n-1)\,a\,T_0}{4}$—in which a is the velocity of propagation of the water hammer wave in feet per second; and T_0 is the period of oscillation of the turbine gates, in seconds.

[53] Cons. Engr., Director, Charles T. Main, Inc., Boston, Mass.

In practice, the value of T_0 will of course have been predetermined by governing stability computations and corresponding selection of physical elements, so as to be far removed from the zone of resonance. However, preserving the original thesis, the lowest conceivable practicable minimum value of $T_0 = 5$ sec and a low value of $a = 3{,}000$ ft per sec will be considered. The first node will occur at a distance $x = \frac{a\,T_0}{2}$ from the reservoir or $x = \frac{3{,}000\ (5)}{2} = 7{,}500$ ft. Consequently, there appears to be no practicable possibility of a node between the reservoir and turbine unless the pressure conduit (without surge tank) exceeds 7,500 ft in length. If so long a conduit were employed without a tank, the governor period would necessarily be made much longer than 5 sec to avoid excessive water hammer; and the valué of x required for the existence of a node would be proportionately increased. It is therefore concluded that in practice the likelihood of a second node between the reservoir and turbine is exceedingly remote.

By analogous reasoning, the peak pressure will decrease steadily (not linearly) from the turbine to the reservoir, unless x exceeds 3,750 ft (without a surge tank); and, if a longer conduit were employed, the value of T_0 would probably be increased as before to reduce water hammer pressures. Therefore, in the majority of cases, there may be expected a steady decrease of peak water hammer values from the turbine to the reservoir in actual commercial installations, even in the event of a hunting governor that actually reaches the resonant condition. However, this in itself is a remote probability, because even in the 1920's no practical operator would have tolerated a swinging governor. He would have slowed down the response of the governor to obtain stability and accepted the resultant sacrifice in regulation.

With reference to Fig. 6(*b*), the writer's investigations lead to the following interpretation. The existence of a complete node at the base of the surge tank indicates not that the surge tank is inoperative, but it definitely shows that the tank is affording maximum benefit—just as the existence of a node at the reservoir does not mean that the reservoir is inoperative, but that it affords infinite relief from both sub-pressures and super-pressures. As an illustration, let a simple tank of infinite size be installed; the tank then becomes a reservoir that (as the author correctly states) will certainly produce a node. If the size of the surge tank is gradually decreased, the node will be replaced by a dip in the pressure curves, the amount of departure from the perfect node being proportional to the decrease in tank size.

An alternative approach is illustrated by the strictly hypothetical case of a very long conduit without a surge tank in conjunction with a very short period of oscillation of the turbine gates—a combination not at all likely to occur for the reasons already given. Under this admittedly artificial system, several nodes could occur in the conduit. If a surge tank of liberal size were then placed over the node nearest the turbine, the author reasons that there would be no effect on the system. The writer's investigations indicate the contrary. The addition of the tank would cause a considerable change in the reflection of the increments generated at the turbine subsequent to the addition of the

tank. Also—(1) The intensity of pressures in the section between the tank and turbine would be greatly reduced; (2) the pressures upstream of the surge tank would be much less in magnitude than those in the downstream penstock; (3) there would be a depression that would at least approach a node at the tank; and (4) the location and number of nodes in the entire system would be changed. In summary, it is contended that the existence of a node at the surge tank is not a detriment or an abnormality; but that it is a benefit that is present, at least in partial form, as a pronounced dip at every tank of adequate size.

To clarify the issues raised by this discussion, it is suggested that the author organize an extensive series of investigations by the electronic computer (using a wide range of values of conduit length and period of gate oscillation) because this topic is of the utmost importance to designers of tanks and conduits.

Regarding surge tank stability, the ambiguity depicted by Fig. 12 applies only to the application of preliminary trial devices such as the Thoma formula. It is the writer's practice always to verify the stability of the tank selected by arithmetic integration, placing a relatively small load on the turbine and making certain that the resultant damping of the surge oscillations is sufficiently rapid—or, in technical language, that the logarithmic decrement is sufficiently large. In such integration, the port or orifice area participates to the correct extent (even though small) and exact account is taken of the shape of the turbine efficiency curve.

In the arithmetic integration for the case of load demand, the turbine gates are always blocked to avoid operation on the rapidly falling part of the efficiency curve. In the writer's experience, it has never been found economical to make the tank or turbine large enough to maintain instantaneous full-load power at the extreme low point of the riser drop curve.

The casual reader will interpret Fig. 14 as a sweeping condemnation of the Thoma formula; although as a matter of fact the Thoma expression was never intended for use except in the zone labeled "Type A—Oscillatory." In this, its intended field of application, there is a remarkable agreement with results obtained by the electronic computer. Zones indicated as "Type B" and "Type C" of the author's Fig. 14 are not usually classified under the heading of stability at all; but are commonly called "draining the tank" and are handled very accurately by arithmetic integration, as well as with good accuracy even by the load-demand charts of R. D. Johnson.[40] Under Zone B, with $\phi < 0.33$ it will be found theoretically possible (perhaps not economical) always to carry full power demand, without draining the tank, by increasing the tank diameter, if the turbine that is purchased is large enough to permit the discharge so required at the reduced head.

In zone "Type C," the conduit friction is so large that the tank action is "dead beat" at the quarter-cycle, so that the tank will drain under full power demand no matter how large the tank is made. In this zone, the gradient change increases more rapidly than the discharge at the constantly dropping head. The standard practice in making arithmetic integration studies in this zone is to block the turbine at about 80% gate, to purchase the turbine only

slightly (if any) larger than necessary to meet the power required at the rated head, and to accept whatever power the turbine can develop under the gradient for the new demanded load at minimum reservoir elevation.

Nevertheless, it would be a great advantage to designing engineers if a more comprehensive and accurate trial chart for preliminary stability tests were made available. Therefore, it is suggested that the author conduct extensive research culminating in a chart similar to Fig. 14. Such a chart would not presuppose constant gate, constant flow, or constant power but would reflect the actual response of the gate and governing mechanism as faithfully as would the arithmetic integration for load demand. In the latter case, the turbine gate openings do not remain constant but move on and off the blocked position in response to head and discharge changes.

As a practicing engineer in the analytical field, the writer can hardly be expected to agree (under the heading, "Surge Tank Problems") that:

> "***The problem of determining the response of tanks under all reasonable disturbances that might be encountered has been neglected in the past, largely because of the altogether forbidding length of time required by even the most elementary investigations using conventional methods."

With twenty or thirty years of background experience with surge problems, most engineers in this field can accomplish a very sizable volume of computation in one or two weeks, for which the consulting costs would be far from forbidding.

In conclusion, it is the opinion of the discusser that the electronic computer can be used to the greatest advantage in pioneering research to provide more incisive tools for the rapid preparation of trial design charts. Such work should be financed by the existing agencies that are organized expressly for the purpose of promoting worthwhile research. This would benefit a broad segment of practicing engineers who are not necessarily specialists in the final surge design, but who nevertheless must always bear the responsibility for the economic coordination of entire projects in which regulation is one component feature. This feature is adjusted by the project engineer in proper economic relation to the other constituent features.

Donald R. F. Harleman[54] and Edward N. Rein,[55] Junior Members, ASCE.—The principal aim of this discussion is to emphasize, further illustrate, and extend the remarks which Mr. Paynter has made about the effect of demand flow characteristics on surge computations. Some additional developments by the author have been utilized to obtain simplified computational procedures for the case of simple surge tanks.[22,56,57] The writers show a comparison of the results of these procedures with some experimental investiga-

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[56] "Regulation of Hydroelectric Plants," by A. T. Gifford and H. M. Paynter (mimeographed notes), 1952.

[57] "New Relationships for the Analysis of Surge Tank Transients," by A. T. Gifford and H. M. Paynter, *Proceedings of the MIT Hydrodynamics Symposium*, Massachusetts Inst. of Technology, Cambridge, Mass., June, 1951.

tions on the surge tank equipment in the M.I.T. Hydrodynamics Laboratory. The Tallulah Falls example is also included for direct comparison with the computer solution.

General Tank Level Equation for Constant Flow.—When Eq. 43 and Eq. 44 are combined, by eliminating the common time variable, the resulting equation is known as the phase or energy equation—

$$\frac{du}{dy} = \frac{y + \frac{1}{2} u^2}{v - u} \quad \text{(51)}$$

which may be rewritten as

$$\frac{dw}{dz} = \frac{z - f}{\Delta R - w} \quad \text{(52)}$$

in which $z \equiv y + \frac{R^2_1}{2}$, $w \equiv u - R_1$, $f \equiv \frac{R^2_1}{2} + \frac{u}{2}\left|u\right|$, and $\Delta R \equiv R_2 - R_1$. These substitutions and $\Sigma R \equiv R_2 + R_1$, $f_0 \equiv \frac{R^2_1}{2} - \frac{R^2_2}{2}$, and $R_2 = v$ (constant demand flow) are shown in Fig. 18.

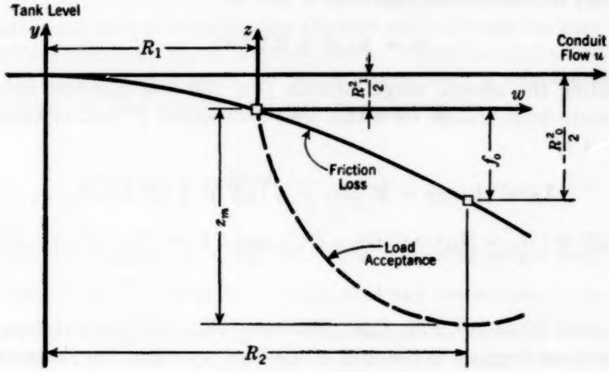

Fig. 18.—Phase Diagram

The solution of Eq. 52 can be obtained by separating variables and integrating, as follows,

$$\frac{z^2}{2} - \int_0^z f\, dz = \frac{(\Delta R)^2}{2} \quad \text{(53a)}$$

in which the value of $\int_0^z f\, dz$ is the area under the surge-friction curve of Fig. 9(*c*). This integral also can be written as

$$\int_0^z f\, dz = K\, z f_0 = -K\, z \frac{\Sigma R\, \Delta R}{2} \quad \text{(53b)}$$

in which K depends only upon the shape of the curve.

Using Eq. 53*b* and solving the resulting quadratic equation for z, the general surge formula for the case of constant demand flow, assuming square-law friction for the simple surge tank, is

$$z_m = -\Delta R\left[\frac{K\,\Sigma R}{2} + \sqrt{1 + \left(\frac{K\,\Sigma R}{2}\right)^2}\right] \qquad (54)$$

To determine values of K, Eq. 53*a* can be solved for z graphically or by the use of the computer. The value of z then can be substituted into Eq. 4, leaving K as the only unknown. Tables of values of K are available.[22]

It has been found that, in the region where R_1 and R_2 approach zero, K approaches a limiting value K_0 which can be expressed as

$$K_0 = \frac{0.33\,R_1 + 0.10\,R_2}{\Sigma R} \qquad (55)$$

Modification of this result, to give values of K that are valid in the range from $R_1 = R_2 = 0$ to $R_1 = R_2 = 1$, leads to the following expression that closely approximates the relationship between K and R:

$$K = K_0\,(1 + 0.1\,\Sigma R) \qquad (56)$$

Translating the general surge formula (Eq. 54) into physical terms, the first extremum level change for either load acceptance or load rejection may be expressed as

$$\text{Level change} = K\,\Delta H_f + \sqrt{(\Delta Y_0)^2 + (K\,\Delta H_f)^2} \qquad (57)$$

in which $\Delta H_f \equiv |H_{f1} - H_{f2}| = C|Q^2_2 - Q^2_1|$; and $\Delta Y_0 \equiv |Y_{02} - Y_{01}| = |Q_2 - Q_1| \sqrt{\dfrac{L_c}{g\,A_c\,A_t}}$.

Surges and Times for Full Load Rejection.—For the computation of the subsequent surge heights, in the case of full load rejection, the phase equation with $v = 0$ becomes

$$\frac{du}{dy} = \frac{y + \frac{1}{2}u|u}{-u} \qquad (58a)$$

for which an approximate solution in physical terms is

$$Y_{i+1} = \frac{\dfrac{3\,Y_0}{2\,R_0}}{\dfrac{3\,Y_0}{2\,R_0\,Y_m} + i} \qquad (58b)$$

in which $i = 1, 2, 3, \ldots$. The sign of the surge alternates appropriately.

The time to the first extreme surge may be computed by using the generalized results of many graphical, computer, and analytical solutions, which are presented in Table 4.

TABLE 4.—TIME TO FIRST EXTREME FOR LOAD REJECTION IN SIMPLE SURGE TANK

Tank parameter, R	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
$\tau_m = t_m/T_{CT}$	0.25	0.26	0.27	0.29	0.30	0.32	0.33	0.35	0.37	0.39	0.41

The period for the subsequent surges of the load rejection case can be assumed to be equal to the free period because of the small effect of friction at low flows. The time between surges can be taken as $t = T_{CT}/2$.

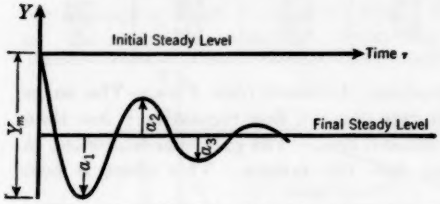

FIG. 19.—SURGE-TIME CURVE

Surges and Times for Load Acceptance, Constant Demand Flow.—The first extreme change in level can be computed from the general surge formula. However, computation of the subsequent surges requires a slightly different technique from that used for the case of load rejection. This is because no generalization can be applied until after the first positive surge has been reached, since quadratic damping has so large an effect. The ratio of the first positive surge to the first extreme surge, in which the symbol a denotes amplitude measured from the final steady level, can be expressed as

$$\frac{a_2}{a_1} = \frac{\dfrac{a_i}{a_{i-1}}}{1 + \dfrac{2}{3} a_1} \qquad (59)$$

Eq. 59 is shown graphically in Fig. 19. The remaining surges may be computed directly from $\frac{a_i}{a_{i-1}}$ which is the ratio that any subsequent surge bears to the surge immediately preceding it. This amplitude ratio is computed by assuming that over a small range of flow near the final steady state, the friction characteristic, is linear. Table 5 shows these amplitude ratios and applies to

TABLE 5.—SURGE AMPLITUDE RATIOS AND HALF PERIODS FOR LOAD ACCEPTANCE IN A SIMPLE SURGE TANK

R_2	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
$\frac{a_i}{a_{i-1}}$	1.000	0.728	0.527	0.373	0.254	0.164	0.095	0.046	0.016	0.002	0.0
$\bar{\tau}_p$	0.500	0.502	0.510	0.524	0.546	0.578	0.625	0.700	0.834	1.148	∞

all surges following the first positive surge for the case of load acceptance and constant demand flow. The half period $\bar{\tau}_p$ is equal to $t_{\frac{1}{2}\,\text{period}}/T_{CT}$.

The time to the first extreme surge for the case of load acceptance and constant demand flow can be computed from the generalized results presented in Table 6, which gives values of $\bar{\tau}_m$. The half periods of the subsequent surges may be computed from Table 5.

TABLE 6.—Time to First Extreme Surge for Load Acceptance

R_2	Q_1/Q_2					
	0.0	0.2	0.4	0.6	0.8	1.0
0.0	0.25	0.25	0.25	0.25	0.25	0.25
0.2	0.26	0.26	0.26	0.26	0.26	0.27
0.4	0.26	0.27	0.27	0.28	0.28	0.29
0.6	0.27	0.27	0.28	0.29	0.30	0.31
0.8	0.28	0.29	0.30	0.31	0.32	0.34
1.0	0.29	0.31	0.32	0.34	0.36	0.39
1.2	0.30	0.32	0.34	0.37	0.40	0.44
1.4	0.31	0.34	0.38	0.42	0.47	0.52
1.6	0.32	0.37	0.42	0.48	0.57	0.66
1.8	0.34	0.40	0.48	0.60	0.76	0.98
2.0	0.37	0.46	0.57	0.73	0.96	∞

Surges and Times for Load Acceptance, Constant Gate Flow.—The surge amplitude that results from constant gate demand flow regulation is less than the surge of a comparable constant demand flow. The gate characteristics, in effect, introduce additional damping into the system. This effect is best

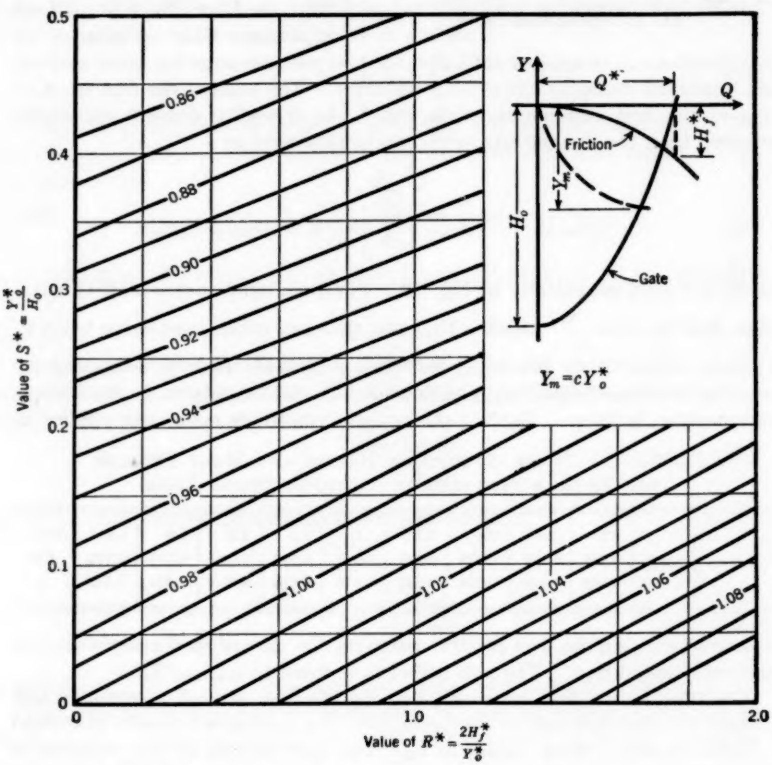

Fig. 20.—Maximum Surge for Load Acceptance with Constant Gate

treated by graphical analysis and the results presented in Fig. 20 permit the computation of the first extreme surge. To account for the gate flow characteristics, a new parameter is introduced and is defined as the ratio of the free surge to the index head. The free surge, in this case, is based on the gate flow that would be obtained under the index head in the absence of friction loss (see Fig. 20). The parameter R^* also is based on this flow. The index head is the total headwater-to-tailwater difference in elevation. The factor obtained from Fig. 20 is then multiplied by the free surge Y^* to obtain the actual surge.

The subsequent surges can be computed in the same manner as those accompanying constant demand flow. However, it is necessary to account for the additional damping. From Fig. 21, the damping action of both

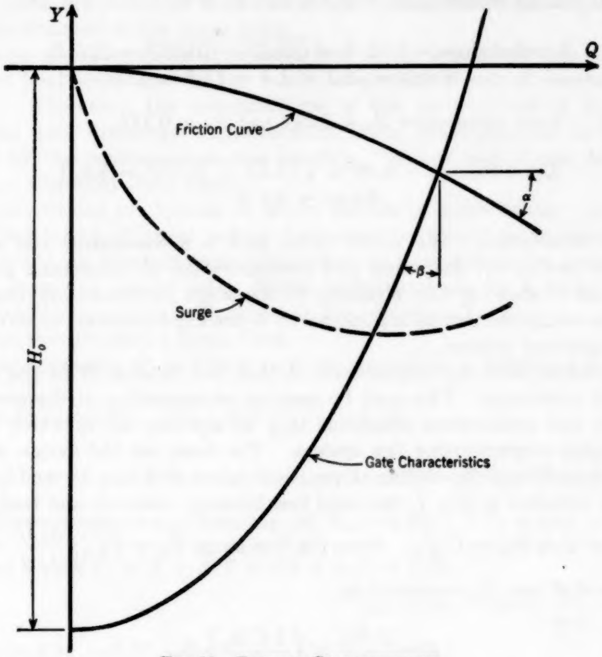

Fig. 21.—Damping Characteristics

friction and gate curves is seen to be dependent on the relative slope of each curve. Therefore, since the curves implement each other, the combined damping is the sum of the tangents of the two angles. This also can be expressed as $R' = R + \frac{S}{2}$. This value would then be used to enter Table 5, so as to select the amplitude ratio and half period.

The time to the first extreme surge should be taken from Table 6 and reduced by an amount dependent on S^*. Graphical analysis shows this amount to be approximately 0.07 S^*. Hence $\bar{\tau}'_m = \bar{\tau}_m - 0.07\ S^*$.

Tallulah Falls Plant, Constant Flow Regulation Assumed.—

Case A.—Load rejection $R_1 = 0.425$ and $R_2 = 0$

$$\text{Level change} = K\,\Delta H_f + \sqrt{(\Delta Y_0)^2 + (K\,\Delta H_f)^2}$$

$$\Delta H_f = 5.0 - 0 = 5.0 \text{ ft} \quad K_0 = \frac{0.33\,R_1 + 0.10\,R_2}{\Sigma R} = 0.33$$

$$\Delta Y_0 = 23.5 - 0 = 23.5 \text{ ft} \quad K = 0.33\,(1 + 0.0425) = 0.344$$

$$\text{Level change} = 0.344 \times 5 + \sqrt{(23.5)^2 + (0.344 \times 5.0)^2} = 25.3 \text{ ft}$$
$$\text{Surge } Y_m = 25.3 - 5.0 = 20.3 \text{ ft}$$

Case B.—Load rejection $R_1 = 0.246$ and $R_2 = 0$

$$\text{Level change} = 0.52 + \sqrt{(12.2)^2 + (0.52)^2} = 12.7 \text{ ft}$$
$$\text{Surge} = 12.7 - 1.5 = 11.2 \text{ ft}$$

Case C.—Load acceptance $R_1 = 0.011$ and $R_2 = 0.111$

$$\text{Level change} = 0.10 + \sqrt{(5.5)^2 + (0.10)^2} = 5.6 \text{ ft}$$
$$\text{Surge} = 5.6 \text{ ft}$$

Model Description.—The model surge tank is a convenient and accurate laboratory facility for the study and demonstration of surge tank problems. As a means of showing the reliability of the surge equations previously presented, the computed results are compared to the experimental values obtained from the physical system.

The general similarity requirement is that the value of R be the same in model and prototype. This may be seen by an inspection of the normalized continuity and acceleration equations (Eq. 43 and Eq. 44) in which R is the only variable characterizing the system. The basis for the design of model equipment capable of simulating all practical values of R may be readily shown. Using the notation of Fig. 7, the head loss between reservoir and tank for the design flow Q_0 is $H_{f0} = C\,Q^2_0$. Since the free surge $Y_0 = V_0\sqrt{\frac{A_c\,L_c}{A_t\,g}} = \frac{Q_0}{A_c}Z_0$, the value of R may be expressed as

$$R = \frac{2\,H_{f0}}{Y_0} = \left(\frac{2\,C\,A_c}{Z_0}\right)Q_0 \qquad (60)$$

Thus the R-value for the model may be made to agree with a prototype value by an adjustment of the conduit flow Q_0 inasmuch as C, A_c, and Z_0 are constants of the equipment.

The vertical scale ratio Y_r for the model tank is determined by the ratio of the head losses for the model and prototype flows. Thus $Y_r = \frac{(H_{f0})_m}{(H_{f0})_p}$ and from the requirement that R be the same in model and prototype, it follows that the vertical scale ratio is also equal to the ratio of the free surge in model

and prototype or $Y_r = \frac{(Y_0)_m}{(Y_0)_p}$. The corresponding time ratio for surges is then equal to the ratio of the free periods, $t_r = \frac{(T_{CT})_m}{(T_{CT})_p}$.

At the M.I.T. Hydrodynamics Laboratory, the model surge tank consists of a rectangular reservoir having a surface area of 14 sq ft, a conduit 49 ft long and 2 in. in diameter, connected to a lucite surge tank 3 in. in diameter and 6 ft high. The penstock, below the surge tank connection, contains the discharge-regulating valve followed by a quick-acting gate valve at the end where the penstock discharges into a sump. From the sump, water is pumped back into the reservoir with provision for matching conduit flow rate and pumping rate in order that a constant total head can be maintained on the system. Data are recorded by marking and timing the surge heights on a paper strip attached to the lucite tank.

Assuming the three types of demand flow shown in Fig. 10, it is seen that the case of load acceptance in the model surge tank is one of constant gate regulation. Therefore, the computations of the performance of this tank include the gate discharge characteristics using the equations and tables presented by the writers under the heading, "Surges and Times for Load Acceptance, Constant Gate Flow."

These conditions are typical of model studies of surge tanks. Since the practical regulation of most actual hydroelectric plants is necessarily for constant power, an inherent error exists when such conditions are present and this error is on the unsafe side. Model indications are for a surge that is smaller than the surge that will appear in the prototype tank.

Computations for Model Surge Tank.—

Load Acceptance.—$Q_1 = 0.0042$ cu ft per sec and $Q_2 = 0.0410$ cu ft per sec

First extreme surge $R^* = \frac{2\,H^*_f}{Y^*_0} = \frac{2\,(0.65)}{1.14} = 1.14$,

and $S^* = \frac{Y^*_0}{H_0} = \frac{1.76}{3.53} = 0.50$

For 90% load acceptance and from Fig. 20, $Y_m = 0.90 \times 1.76 \times 0.87 = 1.38$ ft

Subsequent surges $R' = R + S/2 = 1.0 + 0.22 = 1.22$,

and from Table 5, $\frac{a_i}{a_{i-1}} = 0.090$

$a_1 = 1.38 - 0.77 = 0.61$ $\quad$ $Y_1 = 1.38$ ft

$a_2 = 0.61 \times 0.090 = 0.055$ $\quad$ $Y_2 = 0.77 - 0.06 = 0.71$ ft

$a_3 = 0.055 \times 0.090 = 0.005$ $\quad$ $Y_3 = 0.77 + 0.005 = 0.78$ ft

Time of surge $R_2 = 1$, $\frac{Q_1}{Q_2} = 0.102$, and $T_{CT} = 11.6$ sec

From Table 6, $\bar{\tau}_m = 0.30 - 0.08\,(0.50) = 0.26$, and $t_m = 0.26 \times 11.6 = 3.0$ sec

From Table 5, $\bar{\tau}_p = 0.625$, $t_p = 0.578 \times 11.6 = 6.7$ sec,

and $t_2 = 3.0 + 6.7 = 9.7$ sec

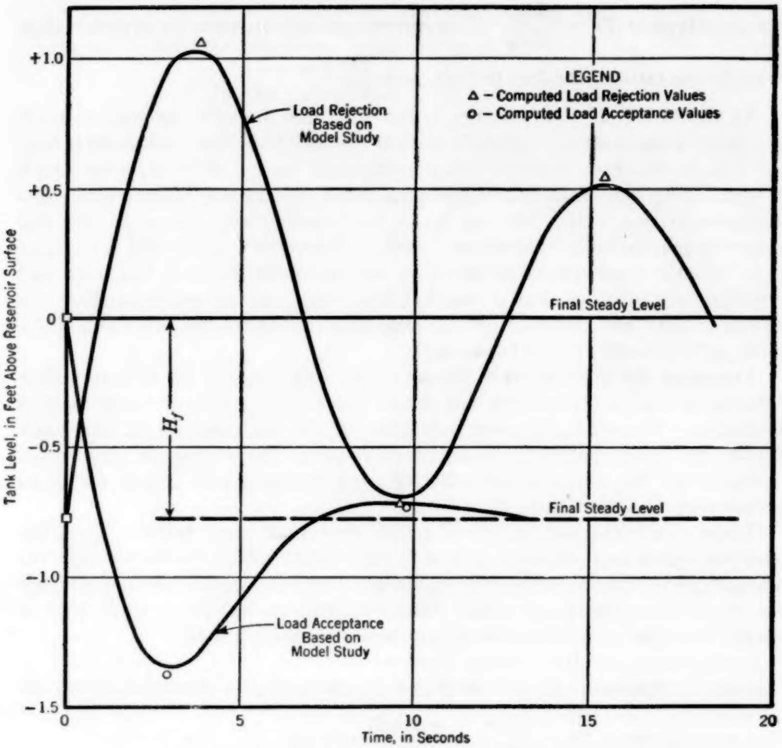

FIG. 22.—TRANSIENTS IN MODEL OF SIMPLE SURGE TANK

Load Rejection.—$R_1 = 1$, $R_2 = 0$, and $Y_0 = 1.54$ ft

First extreme surge

$$\Delta H_f = 0.77 \text{ ft} \qquad K_0 = 0.33$$
$$\Delta Y_0 = 1.54 \text{ ft} \qquad K = 0.33\,(1.1) = 0.364$$

$$\text{Level change} = 0.364 \times 0.77 + \sqrt{(1.54)^2 + (0.364 \times 0.77)^2} = 1.83 \text{ ft}$$

$$\text{Surge} = 1.83 - 0.77 = 1.06 \text{ ft}$$

$$\text{Subsequent surges } Y_{i+1} = \frac{2.30}{2.17 + i},\ Y_2 = -0.725, \text{ and } Y_3 = +0.55$$

Time of surge

From Table 4, $\bar{\tau}_m = 0.32$, $t_m = 0.32 \times 11.6 = 3.7$ sec,

$$t_p = \frac{11.6}{2} = 5.8 \text{ sec},\ t_2 = 9.5 \text{ sec, and } t_3 = 15.3 \text{ sec}$$

Fig. 22 shows a comparison between the surge tank experimental data and the values computed from the formulas.

Henry M. Paynter,[58] J.M. ASCE.—The constructive contributions of the discussers are gratifying to the writer. Although surge tanks and water hammer problems were treated in the paper, the principal subject was the application of electrical analogs and computers to this field. This may account for the fact that much needed explanatory material on specific cases was not included in this particular treatment.

Mr. Strowger is correct in his supposition as to the objectives of the research program on which the paper was based. The main interests were twofold—(*a*) preparation of curves and tables useful in general design and (*b*) detailed treatment of certain special cases, both of isolated plant transients and interconnected system operation. These objectives are outlined in greater detail elsewhere.[59,57] A specific additional objective of this paper has been the demonstration of the value of low-cost, versatile analog computers to engineering firms and other groups concerned with the design, control, or operation of hydraulic, mechanical, and electrical systems.

The Tallulah tank was chosen deliberately because—despite past efforts and opportunities—it has never been possible to account for the measured surge within 1.5 ft out of a total level change of 25 ft, which constitutes a 6% error. This error is fairly typical of what may be expected. It results chiefly from an uncertainty concerning the exact nature of the field conditions.

Nevertheless, Mr. Strowger's contribution of the arithmetic integration solution assuming slow change of demand flow suggests the advisability of outlining certain computed results obtained by formulas not given in the original paper.

When closure is gradual, the time for the surge to reach a peak will be greater than that if the same flow decrement is applied suddenly. This increase in time to the first maximum is approximately one half the effective time of closure. In addition, the maximum level change is reduced by a normally small amount, which varies with the square of the closure time. These two approximations, made through analysis of computed and experimental results, may be expressed as the following equations:

Gradual Closure.—(1) The time to the first maximum is

$$T_{ms} - T_{m0} + 0.5\ T_d \qquad (61)$$

in which T_{ms} is the time to the first maximum for a slow change, T_{m0} represents the time to the first maximum for a sudden change, and T_d is the effective closure time.

(2) The reduction in the surge is

$$\Delta Y = 1.65\ Y_0 \left(\frac{T_d}{T_{ct}}\right)^2 \qquad (62)$$

[58] Asst. Prof. of Hydr. Eng., Dept. of Civ. and San. Eng., Massachusetts Inst. of Technology, Cambridge, Mass.

[59] "Methods and Results from M.I.T. Studies in Unsteady Flow," by H. M. Paynter, *Journal*, Boston Soc. of Civ. Engrs., VXXXIX, No. 2, April, 1952, pp. 120–163.

in which ΔY equals the reduction in the surge height, Y_0 is the free surge for the flow decrement, and T_{ct} represents the free period of the tank and conduit.

For the Tallulah tank, the appropriate numerical calculations (see Table 7) are compared with the results found by Mr. Strowger using arithmetic integration techniques.

TABLE 7.—SLOW-CLOSURE COMPUTATIONS FOR THE TALLULAH TANK

Description	Levels, in feet	Time, in seconds
Constants	$Y_0 = 23.5$	$T_{ct} = 342$; $T_d = 40$
Surge, for sudden closure	$Y_{m0} = 20.0$[a]	$T_{m0} = 92.0$[b]
Corrections	$Y = 1.65 \times 23.5 \times \left(\frac{40}{342}\right)^2 = 0.6$	$\Delta T = 0.5 \times 40 = 20$
Surge, for final closure	$Y_{ms} = 20.0 - 0.6 = 19.4$	$T_{ms} = 92 + 20 = 112$
Mr. Strowger's computations[c]	$Y_{ms} = 19.8$	$T_{ms} = 114$

[a] Above the reservoir water surface. [b] Based on Table 4. [c] See Fig. 17.

Simple corrections may thus be made to the basic formulas to account for significant effects encountered in practice.

Moreover, the computer may account for the variation in tank area with vertical height. Solutions for the most common of these situations have been put into graphical and formula form by the writer and A. T. Gifford.[56]

Mr. Rich questions the engineering importance and physical basis for the examples of elastic resonance outlined in the paper. The cases depicted in Fig. 6 were suggested by a paper by Charles Jaeger.[60] Although Mr. Jaeger asserts at the outset that most cases of resonance occur because of small increments of flow and pressure, for which the flow line is adequately strong, he cites in this reference eleven specific cases in which resonance effects produced serious troubles. In particular, Mr. Jaeger writes:[61]

> "***Resonance is dangerous not only for the pipeline, but sometimes also for the tunnel upstream from the surge tank. It can be theoretically proved that harmonics can penetrate far into a tunnel, in spite of the presence of a surge tank.
>
> "***We have had the opportunity of investigating such a case very thoroughly. Within a long period of operation, extending over many years, fissures had developed three times in a certain tunnel, each time at the same place. Horizontal, parallel fissures, which measured more than 60 feet in length indicated the presence of water hammer. A further examination revealed more fissures at equal distance from one another, and a study of the system showed that there had been a resonance of the eleventh harmonic inside the tunnel.***"

The disturbance producing this frequency was found to be a faulty air valve in the penstock.

[60] "Water Hammer Effects in Power Conduits," by Charles Jaeger, *Civil Engineering and Public Works Review*, Vol. 43, Nos. 500–503, London, England, February–May, 1948.

[61] *Ibid.*, p. 61.

Perhaps the discrepancy between the statements of the writer and the comments of Mr. Rich can be resolved by an examination of the essential behavior of surge tanks, and particularly the difference between (on one extreme) the large surge chamber, forebay, or equalizing reservoir, which completely isolates the conduit from the penstock—thus serving as a wave trap—and in contrast, the ordinary elevated surge tank, stand pipe, or chamber connected to the flow line through a base pipe or riser. In the first case the tank serves as a nodal clamp since the nearly constant pressure condition extends over a length comparable to the wave lengths of all disturbances traveling up the penstock. However, in the second case the tank serves more nearly as a nodal point (rather than a nodal clamp), and the pressure influence is imposed only over a small distance compared to the wave length. Any reader who has had the opportunity to play a stringed instrument (an Hawaiian guitar in particular) will recognize this difference. The tank of the second case may act exactly like the steel bar of the Hawaiian guitar, which serves as a node but permits vibrations of the string on each side.

This description may avoid possible misunderstanding, since the writer had in mind the conditions of the second case whereas Mr. Rich may have been referring to the wave-trap tank of the first type. The writer believes that the analogous situation of capacitor-loaded electrical transmission lines commonly found in practice renders any hydraulic verification of these principles unnecessary; however, systematic analysis and computational research may be in order.

The purpose of Fig. 14 was to emphasize the significance of the Thoma tank-stability criterion for the region of oscillatory instability—both for large load changes in which nonlinear effects are important, and for the small increments that provide the basis for the linear analysis from which the Thoma formula is obtained. The demonstration and proof of this single fact are significant because the required tank diameter for stability as computed by the uncorrected Thoma formula,

$$A_t > A_{thoma}$$

$$A_{thoma} = A_c \left(\frac{V^2{}_0/2\,g}{H_f} \right) \frac{L_c}{H_n}$$

in which H_n is the rated net head equal to $H_0 - H_f$, for all installations in which the friction drop is less than 7% of the static head, can produce at most an error of 7% in the required diameter. In other words, for this oscillatory region, Mr. Rich seems to consider the linear Thoma area (when suitably increased to provide sufficient damping) to be an adequate basis for trial design figures. Nevertheless, if more reliable figures are desired for cases in which the tank will be subjected to large load increments (such as might occur during power-system emergency conditions), the writer suggests his own curve based on analysis and precise computer solutions, in the following form,

$$A_t > A_{0sc}$$

in which $A_{0sc} = \left(1 + \frac{H_f}{H_0}\right)^2 A_{thoma}$ since the Thoma area always errs on the low side.

By contrast, for those fairly uncommon, but nonetheless important, installations in which the value of h_c is greater than 7% but less than 33%, the Thoma formula is entirely misleading for all but the smallest load increments, since a tank based on the Thoma size always will have an incipient tendency to drain for the larger changes in load. Whereas, as Mr. Rich remarks, it may not be economical to design a tank large enough to prevent drainage tendencies for large load increments, the writer would ask only that this situation be appreciated both at the time of design and under operating conditions. For those who may wish to know this drainage limitation, the writer's approximate expression,

$$A_{drain} = 18\, A_c \frac{V^2_0}{2\, H_0} \frac{L_c}{H_0} \qquad (63)$$

indicates the required minimum area to avoid drainage tendencies (or power drop) for the region of h_c between 0.07 and 0.33. It is interesting to note that Eq. 63 does not contain a term representing the conduit friction loss. The two tank areas A_{thoma} and A_{drain} may be compared in computations for a typical hypothetical installation in this range. Data for this example are $L_c = 8{,}000$ ft, $V_0 = 8$ ft per sec, $h_c = 0.20$, $H_f = 20$ ft, $H_0 = 100$ ft, and $V^2_0/2\,g = 1$ ft. Computations for the Thoma area are

$$\frac{A_{thoma}}{A_c} = \frac{L_c}{H_n} \frac{V^2_0}{2\,g\,H_f} = \frac{8{,}000 \times 1}{80 \times 20} = 5.0$$

whereas those for the required minimum area to avoid drainage are

$$\frac{A_{drain}}{A_c} = 18 \frac{L_c}{H_0} \frac{V^2_0}{2\,g\,H_0} = 18 \frac{8{,}000 \times 13}{100 \times 100} = 14.4.$$

From these numbers it is clear that the Thoma area, even if augmented by a considerable allowance, may not satisfy the drainage criterion.

The writer's remarks under the heading, "Surge Tank Problems," were perhaps unfortunately worded. The intent of the paragraph, rather than to disparage the fine work of Mr. Rich, Mr. Strowger, and others in this field, was to affirm the indisputable point that hand-calculation methods of any type are not practically suited to general studies—such as those of the effects of a fluctuating rolling-mill load on the generating units of an interconnected system, and in particular the effects on the level swings of a certain surge tank whose natural period is approximately resonant with the load pulses. Such studies had not been made, to the writer's knowledge, prior to his use of the electronic computer for research of this type. This area of design solutions and exploratory studies is where the newer machine computations and analog methods seem most promising.

The writer wishes to thank Messrs. Harleman and Rein for contributing additional material of later origin, arising from the M.I.T. researches. The bulk of the specific material concerning surge tanks has been placed on a more substantial basis since the original writing of the paper in 1950. Use of these better results for rapid prediction is demonstrated by the discussers. Also, the value of the M.I.T. model tank and its role in the program of computation and analysis is described.

Of particular importance to the hydraulic designer are the specified limitations of the model technique as used at MIT and elsewhere. To overcome the limitations of constant gate operation of the model, a controlled regulating valve can be constructed in order to permit programming any desired relation between demand flow Q_p, total head $H = H_0 + Y$, gate opening B, and time t during a model study. In this way, regulating performance of an actual hydroelectric plant may be simulated by the model tank, in a manner similar to that used with the electronic computer.

Several points in the discussion by Messrs. Harleman and Rein might be clarified herein in order that the data presented may be applied correctly, and that earlier formulas may be appropriately amended.

The expression for the full-rejection surge amplitudes given by Eq. 58*b* corresponds identically to the computational procedure indicated in Table 3, Cols. 1, 2, 3, 5, 6, and 7. Table 4 replaces the formula used to compute the time to the first maximum as given in Table 3, Cols. 4 and 8.

For load acceptance (assuming constant demand flow, which is realistic only if h_c is very small), Eq. 59 and Table 5 replace the graphical procedure indicated in the left-hand part of Fig. 16. The time to the first minimum, as indicated by the first entry in Table 3, Col. 11, may be computed more accurately using Table 6. The remaining time values in this column should be computed from the half-periods given in Table 5.

Fig. 20, which gives values of $c = Y_m/Y^*$, was originally constructed from the results of the electronic computer solutions, and later checked by graphical methods and against model studies. This curve is similar to one provided in the book of J. Calame and D. Gaden.[19] For small values of R^* and S^* these curves may be approximated by

$$c = 1.0 + 0.05\,R^* - 0.35\,S^* \quad \text{..................} \quad (64)$$

For those who wish to see a graphic comparison of actual water behavior as compared to the performance of the electronic computer, contrasting the model tank level measurements of Fig. 22 with the computer oscilloscope photograph of Fig. 11(*a*) may be convincing. Of course, the conditions (the values of R) are different.

In conclusion, the writer wishes to express his opinion as to the importance of finding and using practical methods of analysis and computation in solving engineering problems. In order for any method to qualify as practical for use by the engineer, it must be (*a*) simple and readily understood, (*b*) applicable to varied problems, (*c*) fast, (*d*) inexpensive, (*e*) available, (*f*) adaptable to the use of experimental data, (*g*) free as possible from manipulative errors, (*h*) easily checked, and (*i*) designed to give direct, easily-read results. Of the many tools to which these tests may be applied, three which pass with distinction are numerical methods, graphical methods, and analog methods.

Both Mr. Strowger and Mr. Rich have asserted in their discussions and have demonstrated in their many other writings that any problem involving hydraulic (or other) transients can be solved with simple techniques of arithmetic integration. For the same type of problem, the writer has developed what he considers to be an excellent method of graphical analysis. The

paper attempted to outline the potentialities of analog concepts and analog computers, not only for schools and abstract analysts, but also for operating companies, equipment manufacturers, and consulting firms who have down-to-earth problems in design and development. In both these areas the commercially available, low-cost analog computer can play an important role. However, analog concepts, as distinguished from the actual computers, cost only the time to be learned but may reward the "investor" beyond his expectations.

DISCUSSION OF RAINFALL STUDIES USING RAIN-GAGE NETWORKS AND RADAR
PROCEEDINGS-SEPARATE NO. 178

VEN TE CHOW,[21] A.M. ASCE.—The writer has discussed the problem of

[21] Asst. Prof. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

area-depth relationships for thunderstorm rainfall.[22,23] It was assumed that

[22] Discussion of "Area-Depth Studies for Thunderstorm Rainfall in Illinois," by F. A. Huff and G. E. Stout, by Ven Te Chow, *Transactions*, Am. Geophysical Union, August, 1953.

[23] "Hydrologic Studies of Urban Watersheds, Rainfall and Runoff of Boneyard Creek Watershed, Champaign-Urbana, Illinois," by Ven Te Chow, *Hydraulic Eng. Series No. 2*, Civ. Eng. Studies, Dept. of Civ. Eng., Univ. of Illinois, Urbana, Ill., November 1, 1952.

any observed storm pile can be converted into one which has circular horizontal sections, symmetrical with respect to the vertical axis of the pile. The storm pile is simply a three-dimensional representation of the rainfall isohyetals. By a mathematical treatment, a theoretical general equation for rainfall area-depth distribution for all types of storms was developed. This equation is in the form of an infinite series,

$$H = a + b\,A^{\frac{1}{2}} + c\,A + d\,A^{\frac{3}{2}} + e\,A^2 + \ldots \qquad (5)$$

in which H is the average depth of rainfall on an area A, and a, b, c, d, and e are coefficients.

A further mathematical manipulation shows that Eq. 5 can be expressed as

$$H = \frac{1}{\alpha + \beta\,\sqrt{A}} = (\alpha + \beta\,\sqrt{A})^{-1} \qquad (6)$$

in which α and β are coefficients. By expanding Eq. 6 there results

$$H = \alpha^{-1} + (-1)\,\alpha^{-2}\beta\,A^{\frac{1}{2}} + \frac{(-1)\,(-2)}{1 \times 2}\,\alpha^{-3}\beta^2\,A + \frac{(-1)\,(-2)\,(-3)}{1 \times 2 \times 3}\,\alpha^{-4}\beta^3\,A^{\frac{3}{2}} + \ldots \qquad (7)$$

which is mathematically exactly in the same form as the Eq. 5. The simplified equation can be written as

$$H^{-1} = \alpha + \beta\,\sqrt{A} \qquad (8)$$

which indicates that the reciprocal of H plotted against the square root of area will result in a straight-line relationship for all types of rainfall storms.

If H_0 is the maximum point-rainfall depth measured at the gage site, the ratio of the average rainfall to the point rainfall can be expressed as

$$\frac{H}{H_0} = \frac{1}{1 + \frac{\beta}{\alpha}\,\sqrt{A}} \qquad (9)$$

which is a reduction coefficient to account for a decrease of rainfall depth with increase in drainage area. The value of the ratio β/α is variable and depends mainly on the locality. A study of the data[24] results in an average value of

[24] "Storm Rainfall of Eastern United States," by the Engineering Staff of the District, *Technical Reports*, Miami Conservancy District, Part V, Dayton, Ohio, 1917.

β/α equal to 0.005 (with A expressed in square miles) for the central sections of the United States.

H. E. HUDSON, JR.[25] M. ASCE, G. E. STOUT,[26] AND F. A. HUFF.[27]—The

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[26] Meteorologist, Eng. Subdivision, Illinois State Water Survey, Urbana, Ill.
[27] Meteorologist, Eng. Subdivision, Illinois State Water Survey, Urbana, Ill.

writers are indebted to Mr. Chow for his contributions to the development of the area-depth relationships. They are also indebted to him for a discussion[22] of an earlier paper[28] which is directly related to the present subject. In the

[28] "Area-Depth Studies for Thunderstorm Rainfall in Illinois," by F. A. Huff and G. E. Stout, *Transactions*, Am. Geophysical Union, August, 1952.

earlier paper it was shown that, when area-depth data for thunderstorms were presented as a plotting of average depth of rainfall (H) against the square root of the area (A) within which the rainfall occurs, a linear relation between the two parameters results which can be expressed by Eq. 5, eliminating the higher coefficients c, d, e . . . .

In discussing Eq. 5, Mr. Chow treated the storm pile as a surface of revolution. The analysis revealed that point rainfall is also linearly related to the distance from the center of the storm:

$$y = C + D\,r \qquad (10)$$

in which y is the point rainfall, r is the distance from the center of the storm, and—for a given thunderstorm—C and D are constants. Accordingly, it was concluded that the storm pile was equivalent to a cone of similar dimensions. From this analysis, the writers have determined that $a = C$ and $b = -0.376\,D$, so that

$$H = C - 0.376\,D\,\sqrt{A} \qquad (11)$$

It should be remembered that thunderstorm patterns usually resemble elongated ellipses rather than circles. However, the area-depth data appear to be well correlated when treated as though they were yielded from circular patterns.

It is significant that C is the hypothetical, maximum point rainfall at the center of a storm. The measured maximum point value will probably be smaller than C because of sampling errors and the differences between actual occurrences and analyses. The constant D is, of course, the rainfall gradient, and it is interesting to note that it usually appears to be uniform in nearly all the thunderstorms studied.

Eq. 10 can be used to construct hypothetical area-depth curves for design purposes on small basins when the maximum point rainfall at the storm center can be obtained from frequency data, such as those obtained by D. L. Yarnell,[29]

[29] "Rainfall Intensity-Frequency Data," by D. L. Yarnell, *Miscellaneous Publication No. 204*, U.S.D.A., Washington, D. C., August, 1935.

provided that the rainfall gradient or mean rainfall on the basin can be estimated from a local rain-gage network. The State Water Survey Division of Illinois is at present (1953) investigating the magnitude of thunderstorm rainfall gradients from data collected on small Illinois watersheds in the hope that more concrete information can be obtained on this subject.

Mr. Chow's discussion of relationships in large storms expands the subject. The writers have not had the opportunity to test Eqs. 5 to 9 with data from large storms.

PROCEEDINGS-SEPARATES

The technical papers published in the past twelve months are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

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MAY: 189(HY), 190(HY), 191(CP) & (AT), 192(SM), 193(HY), D-129(PO), D-138(CP), D-145(ST).

JUNE: 194(CP) & (AT), 195(SM), 196(CP) & (AT), 197(HY), 198(ST), 199(EM), D-134(HY), D-141(HY).

JULY: [a] 200(SM)[b], 201(ST)[b], 202(EM)[b], 203(SM)[b], 204(AT)[b], 205(EM)[b], 206(ST)[b], 207(SA)[b], 208(SA)[b], 209(ST)[b], 210(SU)[b], 211(EM)[b], 212(SU)[b], 213(IR)[b], 214(HW)[b], 215(SM)[b], 216(ST)[b], 217(ST)[b], 218(ST)[b], 219(ST)[b], 220(SM)[b], 221(HW)[b], 222(SM)[b], 223(EM)[b], 224(EM)[b], 225(EM)[b], 226(CO)[b], 227(SM)[b], 228(SM)[b], 229(IR)[b].

AUGUST: 230(HY), 231(SA), 232(SA), 233(AT), 234(HW), 235(HW), 237(AT), 238(WW), 239(SA), 240(IR), 241(AT), 242(IR), 243(ST), 244(ST), 245(ST), 246(ST), 247(SA), 248(SA), 249(ST), 250(EM)[c], 251(ST), 252(SA), 253(AT), 254(HY), 255(AT), 256(ST), 257(SA), 258(EM), 259(WW).

SEPTEMBER: 260(AT), 261(EM), 262(SM), 263(ST), 264(WW), 265(ST), 266(ST), 267(SA), 268(CO), 269(CO), 270(CO), 271(SU), 272(SA), 273(PO), 274(HY), 275(WW), 276(HW), 277(SU), 278(SU), 279(SA), 280(IR), 281(EM), 282(SU), 283(SA), 284(SU), 285(CP), 286(EM), 287(EM), 288(SA), 289(CO).

OCTOBER: [d] 290(all Divs), 291(ST)[c], 292(EM)[c], 293(ST)[c], 294(PO)[c], 295(HY)[c], 296(EM)[c], 297(HY)[c], 298(ST)[c], 299(EM)[c], 300(EM)[c], 301(SA)[c], 302(SA)[c], 303(SA)[c], 304(CO)[c], 305(SU)[c], 306(ST)[c], 307(SA)[c], 308(PO)[c], 309(SA)[c], 310(SA)[c], 311(SM)[c], 312(SA)[c], 313(ST)[c], 314(SA)[c], 315(SM)[c], 316(AT), 317(AT), 318(WW), 319(IR), 320(HW).

NOVEMBER: 321(ST), 322(ST), 323(SM), 324(SM), 325(SM), 326(SM), 327(SM), 328(SM), 329(HW), 330(EM)[c], 331(EM)[c], 332(EM)[c], 333(EM)[e], 334(EM), 335(SA), 336(SA), 337(SA), 338(SA), 339(SA), 340(SA), 341(SA), 342(CO), 343(ST), 344(ST), 345(ST), 346(IR), 347(IR), 348(CO), 349(SM), 350(HW), 351(HW), 352(SA), 353(SU), 354(HY), 355(PO), 356(CO), 357(HW), 358(HY).

DECEMBER: 359(AT), 360(SM), 361(HY), 362(HY), 363(SM), 364(HY), 365(HY), 366(HY), 367(SU)[e], 368(WW)[e], 369(IR), 370(AT)[e], 371(SM)[e], 372(CO)[e], 373(ST)[e], 374(EM)[e], 375(EM), 376(EM), 377(SA)[e], 378(PO)[e].

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JANUARY: 379(SM)[e], 380(HY), 381(HY), 382(HY), 383(HY), 384(HY)[e], 385(SM), 386(SM), 387(EM), 388(SA), 389(SU)[e], 390(HY), 391(IR)[e], 392(SA), 393(SU), 394(AT), 395(SA)[e], 396(EM)[e], 397(ST)[e].

FEBRUARY: 398(IR)[f], 399(SA)[f], 400(CO)[f], 401(SM)[f], 402(AT)[f], 403(AT)[f], 404(IR)[f], 405(PO)[f], 406(AT)[f], 407(SU)[f], 408(SU)[f], 409(WW)[f], 410(AT)[f], 411(SA)[f], 412(PO)[f], 413(HY)[f].

MARCH: 414(WW)[f], 415(SU)[f], 416(SM)[f], 417(SM)[f], 418(AT)[f], 419(SA)[f], 420(SA)[f], 421(AT)[f], 422(SA)[f], 423(CP)[f], 424(AT)[f], 425(SM)[f], 426(IR)[f], 427(WW)[f].

APRIL: 428(HY)[e], 429(EM)[e], 430(ST), 431(HY), 432(HY), 433(HY), 434(ST).

a. Beginning with "Proceedings-Separate No. 200," published in July, 1953, the papers were printed by the photo-offset method.
b. Presented at the Miami Beach (Fla.) Convention of the Society in June, 1953.
c. Presented at the New York (N.Y.) Convention of the Society in October, 1953.
d. Beginning with "Proceedings-Separate No. 290," published in October, 1953, an automatic distribution of papers was inaugurated, as outlined in "Civil Engineering," June, 1953, page 66.
e. Discussion of several papers, grouped by divisions.
f. Presented at the Atlanta (Ga.) Convention of the Society in February, 1954.

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